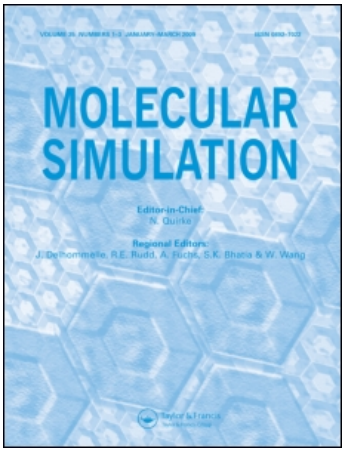

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On the use of fractional charges for computing Fukui functions

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On the use of fractional charges for computing Fukui functions

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Condensed Fukui indices have been computed for a series of organic molecules using fractional charges, as opposed to the conventional approach of performing calculations with the molecular cation and anion. Besides the Fukui function, the eigenvalues, μ, of the highest occupied molecular orbital and lowest unoccupied molecular orbital are also obtained from finite differences. Using a full electron produces errors of up to several hundred percent in the value of μ, and errors of about 5% in atomic Fukui indices. Using a fractional charge of 0.1 can reduce these computational errors significantly.

Keywords: DFT; Fukui function; reactivity; fractional charges

1. Introduction

The chemical potential, chemical hardness and softness and reactivity indices have been used by a number of workers to assess *a priori* the reactivity of chemical species from their intrinsic electronic properties. Perhaps, one of the most successful and best-known methods is the frontier orbital theory of Fukui [1]. Developed further by Parr and Yang [2], the method relates the reactivity of a molecule with respect to electrophilic or nucleophilic attack to the charge density arising from the highest occupied molecular orbital (HOMO) or lowest unoccupied molecular orbital (LUMO), respectively. Parr and co-workers [3,4] were able to use these Fukui indices to deduce the hard-soft acid-bases (HSAB) principle of acid–base interaction from theoretical principles, providing one of the first applications of electronic structure theory to explain chemical reactivity. In essentially the same form, the Fukui functions (FFs) were used to predict the chemical reactivity of a number of systems including Diels–Alder condensations [5,6], mono-substituted benzenes [7], as well as a number of model compounds [8,9]. Recent applications are too numerous to catalogue here but include silylenes [10], pyridinium ions [11] and indoles [12].

Throughout these studies, a variety of molecular orbital methods have been used. Hückel theory was used to study reactions of conjugated molecules [13]; other work used semi-empirical methods to calculate Fukui indices [14]; and a number of workers have used the Hartree–Fock method [5,7–9,15]. Density functional theory (DFT), however, is particularly well suited for performing analyses of this type [16,17]. DFT calculations yield a good representation of the charge density, $\rho(r)$, even in the simplest local-density approximations. This produces reliable results for the chemical properties of molecules and solids. DFT calculations are also fairly computationally inexpensive [18], making this the method of choice for accurate calculations on large molecular and solid-state systems. Finally, DFT programs for periodic systems are now widely available [19], making this the method of choice for modelling chemical reactivity on surfaces or within lattices. Such periodic calculations eliminate the uncertainties introduced by using finite-sized cluster approximations.

In applications like the ones cited above, the magnitude of the FFs are correlated with the reactivity of various sites in a molecule. These FFs can be condensed to atomic-centred indices that can be used to predict which sites are most likely to react with electrophiles or nucleophiles. This can be used to compare the activities of sites within a molecule, or can be used as a measure of how various side groups alter the reactivity of a molecule.

Naturally, the reliability of the predictions will depend upon the accuracy of the method used to compute the Fukui indices. Generally, these calculations are performed by finite differences of the charge density as described in the next section. This paper will discuss aspects of this finite-difference approximation used to compute condensed FFs computed via DFT. In particular, we will illustrate the effects of performing calculations using fractional charges, i.e. non-integer orbital occupations.

2. Theory

A number of quite thorough review papers appear in the literature (see, for example, [17,20,21]), so this section provides only a brief overview of the computation

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of reactivity indices, hardness, and softness. The chemical potential, μ, may be defined as the change in total energy with respect to the number of electrons, N [22]:

$$\mu = \left(\frac{\partial E}{\partial N}\right)_v, \tag{1}$$

where the subscript v indicates that the external potential, i.e. the potential due to the atomic nuclei, is kept fixed. The hardness was defined by Parr and Pearson [3] as:

$$\eta = \frac{1}{2}\left(\frac{\partial^2 E}{\partial N^2}\right)_v \cong \frac{1}{2}(I - A), \tag{2}$$

with I and A being the ionisation potential and electron affinity, respectively. Softness, S, is related to hardness by $S = 1/2\eta$.

One of the first refinements of the technique noted that the derivative with respect to N for a molecular system is necessarily discontinuous [23]: an electron is added to or taken from different orbitals yielding different derivatives. This led to the introduction [2] of the *frontier* or *Fukui function*, $f(r)$, which measures the sensitivity of the charge density, $\rho(r)$, to changes in the number of electrons. The FF f^- measures susceptibility to electrophilic attack (or to loss of electrons),

$$f^-(\vec{r}) = \left(\frac{\partial \rho(\vec{r})}{\partial N}\right)^-_v, \tag{3}$$

while f^+ measures susceptibility to nucleophilic attack (or to gain of electrons),

$$f^+(\vec{r}) = \left(\frac{\partial \rho(\vec{r})}{\partial N}\right)^+_v, \tag{4}$$

The superscript ' $-$ ' refers to the derivative from below and superscript ' $+$ ' to the derivative from above. Equations (3) and (4) are commonly evaluated by adding or removing an electron:

$$f^-(\vec{r}) = \left(\frac{\partial \rho(\vec{r})}{\partial N}\right)^-_v \cong \rho_N(\vec{r}) - \rho_{N-1}(\vec{r}), \tag{5}$$

and

$$f^+(\vec{r}) = \left(\frac{\partial \rho(\vec{r})}{\partial N}\right)^+_v \cong \rho_{N+1}(\vec{r}) - \rho_N(\vec{r}). \tag{6}$$

In this approximation, the charge densities are converged to self-consistency for the original N-electron system, $\rho_N(\vec{r})$, for the system with one extra electron, $\rho_{N+1}(\vec{r})$ and for the electron-deficient system, $\rho_{N-1}(\vec{r})$. The FFs are computed by subtracting the results over a set of grid points and may be displayed as 3D images or 2D contour plots. It has been demonstrated how to compute the FFs analytically for both Hartree–Fock and DFT [15], but finite-difference calculations remain a fixture of the literature owing to the simplicity of their implementation.

Although considerable insight may be obtained from viewing 3D renderings of the FF, it is convenient to make quantitative comparisons among molecules using the condensed FF [15,17,20,24]. For an atom k this is defined as:

$$f_k^- = q(N)_k - q(N-1)_k, \tag{7}$$

and

$$f_k^+ = q(N+1)_k - q(N)_k. \tag{8}$$

The q_k are atomic-centred charges that are computed in some reasonable manner, such as from a Mulliken population analysis [7–9,15], Stockholder analysis [25], fitting the electrostatic potential [25], or numerical integration [20,21]. The term $q(N)$ refers to the atomic charges computed for the original, N-electron system; $q(N+1)$ to charges computed for the system with an additional electron; and $q(N-1)$ to an electron deficient system.

These expressions allow one to compare candidate reactive sites within a molecule as well as analogous sites across a series of compounds. These condensed FFs have been used by a number of authors to compare and predict reactivity for a number of compounds including zeolites [26], Diels–Alder condensations [8] and substituted benzenes [7], to name just a few.

DFT is well-suited for use with non-integer occupations. The Slater transition state formula [27], for example uses half-integer occupations to approximate the electron affinity. Fractional occupations of orbital are also commonly employed in the use of charge smearing to improve self-consistent field (SCF) convergence [28,29].

Using fractional occupations, the partial derivatives are approximated as

$$f^-(\vec{r}) = \left(\frac{\partial \rho(\vec{r})}{\partial N}\right)^-_v \cong \frac{1}{\Delta N}(\rho_N(\vec{r}) - \rho_{N-\Delta}(\vec{r})) \tag{9}$$

and

$$f^+(\vec{r}) = \left(\frac{\partial \rho(\vec{r})}{\partial N}\right)^+_v \cong \frac{1}{\Delta N}(\rho_{N+\Delta}(\vec{r}) - \rho(\vec{r})). \tag{10}$$

The expressions for the condensed FF in Equations (7) and (8) can similarly be written as

$$f_k^- = \frac{[q(N)_k - q(N-\Delta N)_k]}{\Delta N}, \tag{11}$$

$$f_k^+ = \frac{[q(N+\Delta N)_k - q(N)_k]}{\Delta N}. \tag{12}$$

Equations (5)–(8) can be viewed as special cases of these approximations with $\Delta N = 1$. Computing the finite differences with smaller values of ΔN should provide more accurate approximations to the partial derivative. As will be seen below, in many cases the difference is quite startling.

We note that recent work [30] performed a similar analysis not by finite difference of the occupation but by performing finite differences of the external potential by recasting the definition of the FF:

$$f^-(\vec{r}) = \left(\frac{\partial\rho(\vec{r})}{\partial N}\right)_v^- = \left(\frac{\delta\mu}{\delta v(r)}\right)_N^-. \tag{13}$$

Although the method provides reliable results, the authors noted that the method has a disadvantage of a relatively high cost.

3. Computational details

To test the impact of non-integer ΔN on FFs, DFT calculations were performed on a total of 23 molecules. All DFT calculations were performed using the program DMol^3 [18,19], which is already configured to perform calculations with fractional numbers of electrons. The calculations employed a double-numerical basis with *d*-functions on second row atoms (DND basis; [18]), a medium numerical integration grid (yielding an average of 1600 points/atom) and Vosko Wilk, Nusair (VWN) Local Density Approximation (LDA) exchange-correlation [31]. All molecular geometries were optimised at this level of theory.

FFs and condensed FFs were computed by finite-differences with $\Delta N = 0.01$, 0.1 and 1.0. For each system, the DFT energy was converged to self-consistency. Atomic point charges were computed as described below. The SCF calculation was repeated using charges of $\Delta N = 0.01$, 0.1 and 1.0, and the atomic point charges were again computed, and condensed FFs were evaluated via Equations (11)–(12).

Recently, Ayers et al. [20] used sound mathematical arguments to recommend using Hirshfeld partitioning to compute the charges, and in our approach such atomic charges were evaluated by numerical integration. This is similar to the approach used by Gilardoni et al. [21], who used Becke's [32] 3D integration scheme and partitioned the space into atomic sub-regions. In the case of DMol^3, the program employs partition functions to divide space into regions associated with an atomic centre. Atomic charges were computed by integrating the charge density over all grid points while applying an appropriate partition function:

$$q_k^{\mathrm{H}} = Z_k - \sum_i \rho(\vec{r}_i)\frac{\rho_k(\vec{r}_i)}{\rho_T(\vec{r}_i)}, \tag{14}$$

where Z_k is the nuclear charge and $\rho_k(\vec{r}_i)$ is the charge density of the isolated atom k, and

$$\rho_T(\vec{r}_i) = \sum_j \rho_j(\vec{r}_i). \tag{15}$$

The sum over i runs over all numerical integration points in the molecule and the sum over j includes all atoms. This yields the Hirshfeld atomic charges. We designate this type of atomic charge as q_k^{H} and will refer to these as Hirshfeld charges throughout the paper.

4. Finite-difference computation of the orbital eigenvalues

Using fractional charges, it is illustrative to investigate how well Equation (1) can be approximated by finite-difference calculations. This provides a quantitative and unambiguous method for estimating the errors introduced by using various fractional charges, ΔN. We can approximate Equation (1) as:

$$\mu_i \cong \frac{\Delta E}{\Delta n_i}. \tag{16}$$

Here μ_i is the eigenvalue of orbital i and n_i is its occupation number. This is a finite-difference approximation to Janak's theorem [22], so one naturally expects smaller values of ΔN to yield more precise values. Table 1 summarises the actual HOMO and LUMO values together with those computed using the approximation in Equation (16) for a total of 23 molecular systems. In general, using a full electron charge, $\Delta N = 1.0$, yields very poor results: average root-mean square (RMS) errors of -67 and 200% for the HOMO and LUMO, respectively. By contrast, $\Delta N = 0.01$ produces excellent agreement: average RMS errors of -0.4 and 1.7% for the HOMO and LUMO, respectively. As would be expected, the intermediate value, $\Delta N = 0.1$, yields RMS errors between these two extremes: -6.4 and 19%.

Although $\Delta N = 0.01$, yields a more accurate approximation to the analytical derivative, there is a practical consideration. Employing $\Delta N = 0.01$ induces very small changes in the energy and charge density; hence, the SCF must be converged extremely well in order to apply Equation (16). As a compromise between precision and the number of SCF iterations, we have selected $\Delta N = 0.1$ as our 'standard' approach.

Table 1. Percent error in computed eigenvalues when approximating Equation (1) by finite difference.

			(VWN, DND, $\Delta N = 1.0$)		(VWN, DND, $\Delta N = 0.1$)		(VWN, DND, $\Delta N = 0.01$)	
Molecule	HOMO (au)	LUMO (au)	HOMO error (%)	LUMO error (%)	HOMO error (%)	LUMO error (%)	HOMO error (%)	LUMO error (%)
C_6H_5F	−0.2377	−0.0619	−49.5	180.9	−4.9	18.1	−0.5	1.6
C_6H_5CN	−0.2619	−0.1028	−41.0	95.9	−4.1	10.0	−0.4	1.0
$C_6H_5NH_2$	−0.1814	−0.0384	−61.5	284.2	−5.9	28.7	−0.4	3.1
C_6H_5OH	−0.2099	−0.0674	−55.4	164.1	−5.4	16.6	−0.4	1.6
$CH_2CHCHCH{-}CH_3$	−0.2483	−0.1276	−44.9	83.5	−4.5	8.4	−0.4	0.8
$CH_2CHCHCH{-}CN$	−0.2424	−0.1192	−45.3	88.2	−4.4	8.9	−0.4	0.9
$CH_2CHCHCH{-}COOH$	−0.1664	−0.0477	−62.8	208.7	−5.9	22.1	−0.2	2.5
$CH_2CHCHCH{-}N(CH_3)_2$	−0.1858	−0.0682	−61.0	157.0	−5.9	16.0	−0.4	1.6
$CH_2CHCHCH{-}OCH_3$	−0.2275	−0.1253	−69.8	100.4	−6.6	10.0	−0.2	0.8
$C_2H_3{-}CHO$	−0.2786	−0.1113	−48.5	113.1	−4.8	11.2	−0.5	0.9
$C_2H_3{-}COOCH_3$	−0.2530	−0.1048	−62.0	117.5	−6.1	11.7	−0.2	1.0
$C_2H_3{-}NO_2$	−0.1735	−0.0482	−66.8	223.8	−6.4	23.4	−0.4	2.5
$C_2H_3{-}NH_2$	−0.0705	−0.0484	−169.9	197.9	−16.4	23.9	−1.3	2.1
Pyridine	−0.2188	−0.0858	−57.4	133.8	−5.7	13.3	−0.5	1.0
Butadiene	−0.1563	0.0535	−110.3	79.7	−10.7	−14.4	−0.8	−1.7
Ethylene	−0.2547	−0.0406	−64.0	344.8	−6.3	34.2	−0.7	2.2
H_2CO	−0.2343	−0.1039	−78.9	156.0	−7.4	15.5	−0.3	1.0
H_2O	−0.2606	−0.0076	−95.0	1440.5	−8.9	128.2	−0.7	13.2
Maleic anhydride	−0.2741	−0.1711	−48.4	69.7	−4.5	6.9	−0.1	0.6
Maleimide	−0.2492	−0.1534	−51.8	76.2	−4.8	7.5	−0.1	0.6
cis-acrolein	−0.2274	−0.1252	−69.9	100.5	−6.6	9.9	−0.2	0.7
trans-acrolein	−0.2274	−0.1207	−69.4	105.4	−6.6	10.3	−0.2	0.7
NH_2OH	−0.2786	−0.1111	−48.5	113.4	−4.8	11.3	−0.5	0.9
Average % RMS			−66.6	201.5	−6.4	18.8	−0.4	1.7

5. Results for f^+ and f^-

This section discusses the effects on the FFs of the fractional charge, ΔN. Condensed FFs f^+ and f^- were computed for these same of 23 molecules at the geometries optimised with the DND basis and VWN potential. Table 2 lists the RMS differences relative to the VWN/DND/$\Delta N = 0.1$ results for $\Delta N = 0.01$ and $\Delta N = 1.0$. Many of the atomic FFs are quite small, on the order of 0.01–0.05. Consequently, very small differences in the results can inflate the RMS values. For this reason, the results in Table 2 comprise only the 'chemically significant' FFs, which are arbitrarily taken as f^+ or $f^- > 0.10$.

Decreasing the fractional charge, ΔN, results in an improved approximation to the derivatives in Equations (3) and (4), but this does not yield a significant change in f^- and f^+. Table 2 shows that the differences between $\Delta N = 0.1$ and $\Delta N = 0.01$ are less than 1% for f^- and f^+.

Most of the published literature on FFs employs a charge difference of $\Delta N = 1.0$. (An exception for example, is work that compared FFs from relaxed Kohn–Sham orbitals to finite difference calculations extrapolated to $\Delta N \rightarrow 0$ [33].) As seen in Table 2 results using $\Delta N = 1.0$ differ from the more accurate numerical derivatives by about 4–5%, on average. Given the relatively qualitative nature of the FFs, this would normally be adequate for most work. Note, however, that the average RMS for all atoms, not just the chemically significant ones, is about 9% for f^- and 20% for f^+, so the use of $\Delta N = 1.0$ becomes more pronounced for smaller values. We also note that several of the systems experienced difficulty with SCF convergence which is less likely with smaller ΔN.

Table 2. Percent RMS error for atomic FFs in a molecule compared to the (VWN/DND/$\Delta N = 0.1$) results.

	(VWN, DND, $\Delta N = 1.0$)		(VWN, DND, $\Delta N = 0.01$)	
Molecule	f^-	f^+	f^-	f^+
C_6H_5F	3.0	1.6	0.0	0.0
C_6H_5CN	1.7	0.8	0.0	0.4
$C_6H_5NH_2$	4.8	0.8	1.0	0.0
C_6H_5OH	2.8	1.5	0.0	0.0
$CH_2CHCHCH{-}CH_3$	3.7	3.6	0.4	0.0
$CH_2CHCHCH{-}CN$	3.4	4.4	0.5	0.6
$CH_2CHCHCH{-}COOH$	9.8	4.5	6.0	0.9
$CH_2CHCHCH{-}N(CH_3)_2$	2.5	14.4	0.5	0.8
$CH_2CHCHCH{-}OCH_3$	4.5	3.6	0.7	0.5
$C_2H_3{-}CHO$	7.4	3.7	0.7	0.7
$C_2H_3{-}CN$	1.7	3.2	0.4	0.5
$C_2H_3{-}COOH$	13.2	14.2	1.1	0.5
$C_2H_3{-}NH_2$	3.1	14.2	0.6	0
Pyridine	1.9	29.9	0.	0.5
Butadiene	1.4	1.7	0.0	0.0
Ethylene	1.7	3.3	0.0	0.2
H_2CO	2.9	3.7	0.2	0.5
H_2O	4.9	0.8	0.5	0.7
Maleic anhydride	3.2	1.0	0.0	0.0
Maleimide	3.6	1.4	0.0	0.0
cis-acrolein	7.4	3.4	0.6	0.6
trans-acrolein	6.9	3.5	0.7	0.6
NH_2OH	9.8	11.7	1.3	5.3
Average % RMS	4.6	5.7	0.7	0.6

Table 3. Comparison of computed FFs for heavier atoms with previous work.

	This work		Gilardoni et al. [21]	
Atom	f^-	f^+	f^-	f^+
H_2CO				
O	0.40	0.30	0.46	0.24
C	0.24	0.39	0.18	0.40
H	0.18	0.16	10.8	0.18
Maleic anhydride*				
C_a	0.08	0.10	0.08	0.09
C_b	0.07	0.14	0.06	0.13
Maleimide*				
C_a	0.08	0.10	0.07	0.09
C_b	0.07	0.13	0.06	0.13
NH_2OH				
O	0.26	0.16	0.24	0.26
N	0.40	0.18	0.40	0.08
H(O)	0.11	0.26	0.11	0.38
H(N)	0.12	0.21	0.13	0.14

* See [21] for definitions of C_a and C_b.

A comparison of the FFs computed by this procedure to literature values is not straight-forward since there are significant differences in the basis sets and some differences in the numerical integration. Nevertheless, results for selected atoms were compared with earlier DFT results [21], using $\Delta N = 1.0$ for consistency. Despite the many differences in the computational parameters, the results listed in Table 3 are remarkably similar. This validates the results computed using DMol[3] with an external baseline.

Based on these molecular results for the FFs and the finite-difference results for μ_i, we recommend using partial charges of $\Delta N = 0.1$. This has actually been set as the default value for FF calculations in the commercial DMol[3] program.

6. Summary

For the first time, to our knowledge, this work has illuminated the advantage of fractional occupation on the computed Fukui indices and the chemical potential μ. In order to compute the DFT μ defined in Equation (1) to within a few percent, a value of $\Delta N = 0.01$–0.1 is required. Using a full electron, $\Delta N = 1.0$, produces errors of up to several hundred percent in the value of μ, but errors of only $\sim$5% in atomic Fukui indices.

These calculations demonstrate that using fractional charges offers a clear advantage in terms of the numerical precision of the calculations. It also provides a speed advantage, since the SCF calculations converge very quickly when starting from the results for the neutral molecule.

Subsequent work will explore the effects of fractional occupation in periodic systems such as zeolite for both cluster models and periodic systems.

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